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A GENERALIZED TWO-PARAMETER RELATION FOR CONDUCTION-CONTROLLED REWETTING OF A HOT VERTICAL SURFACE

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NOMENCLATURE

B, Biot number, $h\delta/k$;
$\bar{B}$, modified Biot number, equation (9);
c, specific heat of solid;
h, heat-transfer coefficient;
k, thermal conductivity of solid;
P, Peclet number, $\rho c u\delta/k$;
$\bar{P}$, modified Peclet number, equation (8);
$\bar{T}$, dimensionless temperature, $\theta_0^{1/2}/(1-\theta_0)$;
T_0, sputtering temperature;
T_s, saturation temperature;
T_w, initial dry wall temperature;
u, wetting front velocity;
δ, one-half slab thickness;
θ_0, dimensionless temperature, $(T_w - T_0)/(T_w - T_s)$;
ρ, density of solid

1. INTRODUCTION

SURFACE rewetting refers to the establishing of liquid contact with a solid surface whose initial temperature is higher than the sputtering temperature. Due to its application to the emergency core cooling of water reactors in the event of postulated loss-of-coolant accidents, the problem of surface rewetting has gained much attention in recent years. In addition to the one-dimensional analyses [1, 2], recent two-dimensional analyses on this problem include the analytical study by Tien and Yao [3], and the numerical studies by Andersen and Hansen [4], Coney [5] and others [6]. The physical model commonly employed [1, 3–5] consists of an infinitely extended vertical surface with the dry region ahead of the falling film treated as adiabatic and the wet region behind the falling film associated with a constant heat-transfer coefficient.

The present work is a sequel to the analytical study by Tien and Yao [3] wherein asymptotic solutions, for the limiting cases of small and large Peclet numbers, were obtained by utilizing the Wiener–Hopf technique with appropriate kernel substitutions. They characterized the physical problem by three dimensionless parameters: the Peclet number or the dimensionless wetting velocity, the Biot number and a dimensionless temperature. However, on the basis of their numerical results. Andersen and Hansen [4] suggested an empirical relationship between two dimensionless parameters, namely, the modified Biot and Peclet numbers. The reduction of governing dimensionless groups from three to two constitutes a significant simplification for practical applications. It is the purpose of this work to demonstrate that a similar relation in terms of the modified Biot and Peclet numbers can be derived directly from the analysis of Tien and Yao. Moreover, a simple continuous relation is established for the entire range of Biot numbers by empirically matching the asymptotic expressions.

2. ANALYTICAL DEDUCTION OF A TWO-PARAMETER RELATION

The basic mathematical problem, applicable to rewetting, is to seek the solution to transient two-dimensional conduction equation, with the coordinate system moving with the wet front at a constant velocity u, employing appropriate boundary conditions. The details of the mathematical problem are described elsewhere [3]. For the limiting case of both the Peclet and the Biot numbers being small, the result approaches to the one-dimensional result [1, 2]:

$$\frac{B^{1/2}}{P} = \frac{\theta_0^{1/2}}{(1-\theta_0)}. \quad (1)$$

For large Peclet numbers, the two-dimensional asymptotic solution is given as [3]

$$\frac{B}{P} = 1.707\left(\frac{\theta_0}{1-\theta_0}\right) + 1.457\left(\frac{\theta_0}{1-\theta_0}\right)^2. \quad (2)$$

By defining $\bar{T} = \theta_0^{1/2}/(1-\theta_0)$ and $P_0 = P\bar{T}$, equations (1) and (2) are modified as given below:

$$P_0 = B^{1/2} \quad \text{(for small } B) \quad (3)$$

$$P_0 = f(\bar{T})B \quad \text{(for large } B) \quad (4)$$

where

$$f(\bar{T}) = \frac{\bar{T}}{(1.707/2)[(1+4\bar{T}^2)^{1/2}-1] + (1.457/4)[(1+4\bar{T}^2)^{1/2}-1]^2}.$$

The essential step to eliminate the explicit dependence of equation (4) on $\bar{T}$ is to express $\bar{T}$ as a function of the Biot number. The intersections of the lines given by equations (3) and (4) determine $f(\bar{T})$ as a function of the Biot number. Thus at the intersection points,

$$f(\bar{T}) = B_0^{-1/2} \quad (5)$$

where B_0 is the value of the Biot number at which one and two-dimensional solutions intersect.

There exists no simple analytic solution to equation (5). In the limits of $\bar{T} \ll 1$ and $\bar{T} \gg 1$, however, the solution to equation (5) is given as:

$$\ln \bar{T} = -0.54 + 0.50 \ln B_0 \quad (\bar{T} \ll 1)$$

$$\ln \bar{T} = -0.38 + 0.50 \ln B_0 \quad (\bar{T} \gg 1).$$

Of practical interest is the range of $\bar{T}$ from 0.5 to 4.0 wherein most of the available experimental data exist [1, 7]. In view of this intermediate range as well as the limiting forms, it is logical to employ an approximate solution for the entire range of $\bar{T}$ as

$$\ln \bar{T} = -0.46 + 0.50 \ln B_0 \quad (6)$$

where the constant term is the average of the two constants in the limiting forms. Equation (5) can be solved graphically by plotting $\ln \bar{T}$ as a function of $\ln B_0$ and the graphical solution thus obtained also agrees very well with equation (6). It is interesting to note that Andersen and Hansen [4] obtained equation (4) as a result of their numerical calculations and proposed the following empirical relation as the solution to equation (5)

$$\ln \bar{T} = -0.3466 + 0.5642 \ln B_0. \quad (7)$$

To proceed further to eliminate the explicit parametric dependence in $\bar{T}$ or θ_0 in equations (1) and (2), the transformations as suggested by Andersen and Hansen [4]

$$\bar{P} = P\bar{T}^{(2m-1)/2m} \tag{8}$$

$$\bar{B} = B\bar{T}^{(-1/m)} \tag{9}$$

are employed here where m is the coefficient of the ($\ln B_0$)-term in equations (6) or (7). In the present case ($m = 0.5$), $\bar{P} = P$ and $\bar{B} = B\bar{T}^{-2}$, therefore equations (1) and (2) become:

$$P = \bar{B}^{1/2} \quad \text{(for small } \bar{B}\text{)} \tag{10}$$

$$P = 0.63\bar{B} \quad \text{(for large } \bar{B}\text{)}. \tag{11}$$

Equations (10) and (11) represent a generalized two-parameter relation, valid for all values of θ_0. It should be emphasized here that in equations (10) and (11), only the Biot number needs to be modified without having to modify the Peclet number. This provides further simplification and convenience in practical applications.

Analogous to equations (10) and (11), the correlations proposed by Andersen and Hansen [4] based on their numerical calculations are:

$$\bar{P} = \bar{B}^{1/2} \quad \text{(for small } \bar{B}\text{)} \tag{12}$$

$$\bar{P} = 0.7355\bar{B} \quad \text{(for large } \bar{B}\text{)} \tag{13}$$

where $m = 1/\pi^{1/2} = 0.5642$ has been used in equations (8) and (9), defining the transformations in conjunction with the results in equations (12) and (13).

3. A SIMPLE CONTINUOUS TWO-PARAMETER RELATION

Equations (10) and (11) contain the limiting relations for either very small or very large values of the Biot number. However, in practice, the Biot number encountered most likely falls in the intermediate range [6, 7]. Therefore, it is desirable to establish a continuous expression for the entire range of B. The semi-empirical matching

$$P = \bar{B}^{1/2}[1+(0.63\bar{B}^{1/2})^{1/n}]^n$$

provides a family of curves corresponding to different values of n, which satisfy both the limits in equations (10) and (11) and allows a smooth continuous two-parameter relation for all values of $\bar{B}$. In particular, the relation with $n = 1/2$

$$P = [\bar{B}(1+0.40\bar{B})]^{1/2} \tag{14}$$

is not only extremely simple, but also agrees well with the numerical solution [4]. In Fig. 1, equation (14) is shown along with the limiting solutions contained in equations (10) and (11), as well as the numerical solution of Andersen and Hansen [4].

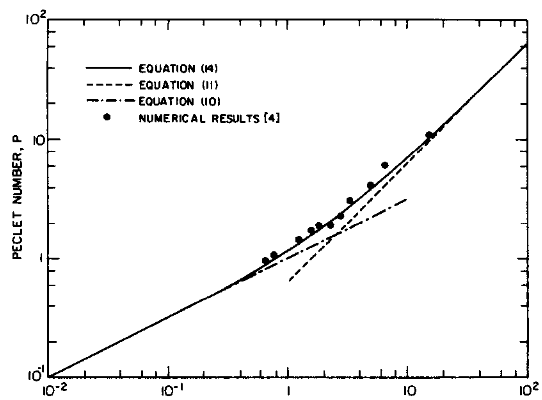

FIG. 1. Variations of the modified Peclet number with respect to the modified Biot number.

4. RESULTS AND DISCUSSION

As demonstrated above, it is possible to get a generalized two-parameter relation from the analytic study of Tien and Yao [3] analogous to the one obtained by Andersen and Hansen [4] based on their numerical calculations. Coney's numerical results [5] do not, however, yield a single correlation for all values of θ_0. In Fig. 2 is shown a comparison of numerical results of Andersen and Hansen [4] and Coney [5] with the asymptotic two-parameter relations, equations (10) and (11), presented here. The comparison is drawn for $\bar{T} = 0.1$, 1.0 and 10.

Equation (10), for the limiting case of very small $\bar{B}$, is a well-established one-dimensional solution confirmed by all the quoted studies [1–5]. Equation (11), for the limiting case of very large $\bar{B}$, agrees well with the numerical calculations of Andersen and Hansen at $\bar{T} = 0.1$, whereas Coney's numerical results predict slightly lower values of the Peclet number at this temperature. At $\bar{T} = 1.0$, equation (11) agrees well with Coney's numerical calculations whereas Andersen and Hansen's correlation predicts slightly higher values of the Peclet number.

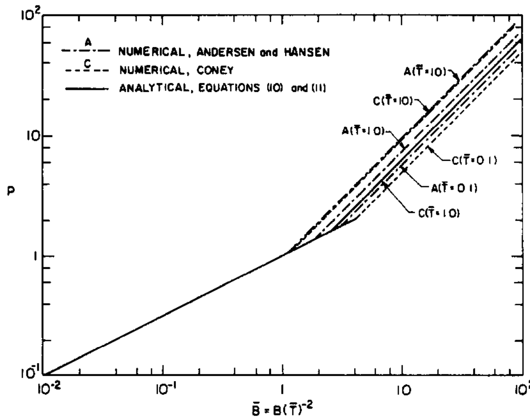

FIG. 2. Comparison between the present two-parameter relation and the numerical results.

At $\bar{T} = 10$ (or $\theta_0 = 0.905$), Coney's numerical results match quite closely with Andersen and Hansen's correlation but both the numerical studies predict higher values of the Peclet number than those predicted by the present two-parameter relation. As seen from Fig. 2 of the previous work [3], it is not possible to discover the true asymptotic relation at $\theta_0 = 0.9$ unless the numerical calculations are carried on to very high values of the Biot number (at least larger than 700 or so). Coney carried on numerical calculations up to $B = 100$; whereas Andersen and Hansen considered a maximum of $B = 1453.5$ for their calculations at $\bar{T} = 9.487$ (or $\theta_0 = 0.9$), but then this ($B = 1453.5$) was the only one point in their calculations which was greater than $B \simeq 700$, and it is conceivable that one point cannot recover the true asymptotic relation. It must, however, be emphasized here that even if the numerical calculations are carried on to high enough Biot numbers, the true asymptotic solution may still not be discovered due to increasingly large truncation error.

How truly an asymptotic relation can be discovered obviously depends upon how appropriately the range of the Biot number, corresponding to a given value of θ_0, is chosen in numerical calculations. The error on account of this, rather arbitrary, discretion is eliminated in the present generalized two-parameter relation due to its analytical extraction in the two asymptotic limits.

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BOILING OF LIQUID NITROGEN AND METHANE ON WATER. THE EFFECT OF INITIAL WATER TEMPERATURE

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DRAKE *et al.* [1] recently reported the rates of vaporization of liquefied cryogens spilled on a water surface. Primary emphasis was placed on the determination of boiling rates. Vapor temperatures were also monitored but few measurements were made in the bulk water. Mapping of liquid water temperatures must, however, be carried out before realistic heat-transfer models can be developed.

The purpose of the present note is, therefore, to report the results of additional boiling studies that were carried out in a vessel equipped to measure the temperature–time history at a number of locations in the bulk water phase.

The apparatus employed was similar to that described in [1] and consisted of an insulated container about 10 cm in diameter. The major change, as noted above, was the deployment of six fast-response thermocouples in the water. As before, one was placed so that it was close to the cryogen–water interface. The thermocouple locations were varied to sample a number of sites. The spill procedure was identical to that described in [1]; however, the data were recorded with a NOVA 840 real-time computer so rapid sampling ($10\,s^{-1}$) could be achieved. Values averaged over a 1-s period were recorded.

Boil-off rates were also determined and these agreed well with those reported previously [1]. Also, as noted in this earlier study, nitrogen vapor was significantly superheated whereas, for liquid methane, boil-off vapors were close to the expected saturation temperatures. Quantitative data are given elsewhere [2].

Of more interest here are the liquid water temperatures. For spills of both liquid nitrogen and liquid methane, it was verified that the vaporization rates were independent of initial water temperature—at least over the range of 6–42°C studied here. However, there was a significant difference in the pattern of water temperatures depending upon whether the water, initially, was warm or cool.

In experiments where the initial water temperature was below 20°C, coherent ice formed almost immediately on the surface and water temperatures beneath this sheet changed but little during a test. In Fig. 1, we show temperature–time traces for the six thermocouples during a spill of liquid nitrogen on 6.6°C water. There is a significant drop in temperature for the thermocouple 0.4 cm below the interface but at all other locations little variation is noted. An energy balance for this run indicated that the ice layer thickness at the end of the test was greater than 0.5 cm.

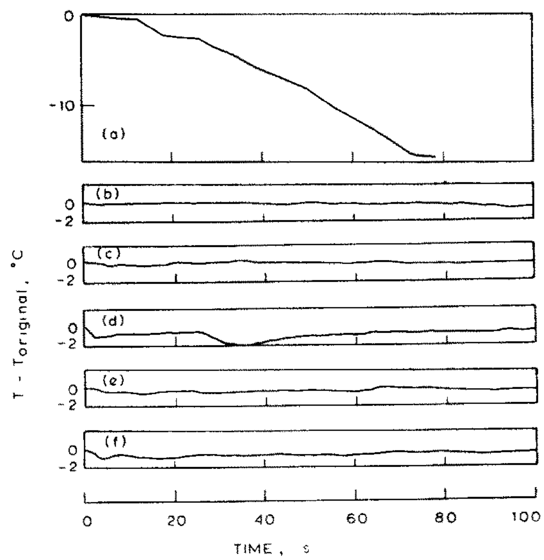

FIG. 1. Water temperatures after a spill of liquid nitrogen on 6.6°C water. 0.94 g/cm^2 of nitrogen was spilled. Thermocouple locations measured below the interface were as follows: (a) 0.4 cm, center; (b) 5.5 cm, center; (c) 3.2 cm, center; (d) 1.4 cm, side; (e) 3.5 cm, side; (f) 4.2 cm, side.

If the initial water temperature exceeded about 25°C, surface ice formed, but quite slowly. The water temperature in all locations decreased almost uniformly, although the largest decrease always occurred in the immediate vicinity of the interface. In Fig. 2, temperature–time traces are shown for a liquid nitrogen spill on 42.5°C water. For the tests shown in Figs. 1 and 2, the amount of liquid nitrogen spilled was the same and the boil-off rates were virtually identical. Similar water temperature observations were noted in other experiments with both liquid nitrogen and liquid methane.

From these experiments it is concluded that if the initial water temperature is low, heat transfer to the cryogen occurs through a growing ice shield with little effect on the underlying water. On the other hand, if the water is initially warm, ice forms more slowly and cool surface water convectively